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Fixed Point Theorem in Complete Fuzzy Metric Space

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ABSTRACT: In this paper our works establish a new fixed point theorem for a different type of mapping in complete fuzzy metric space. Here we define a mapping by using some proved results and obtain a result on the actuality of fixed points. We inspired by the concept of Hossein Piri and Poom Kumam [15]. They introduced the fixed point theorem for generalized F-suzuki -contraction mappings in complete b-metric space. Next Robert plebaniak [16] express his idea by result "New generalized fuzzy metric space and fixed point theorem in fuzzy metric space". This paper also induces comparing of the outcome with existing result in the literature.

Keywords: Fuzzy set, Fuzzy metric space, Cauchy sequence Non- decreasing sequence, Fixed point, Mapping.

I. INTRODUCTION

The thought of fuzzy metric space is given by numerous authors and they explain the fixed point theorem in unrelated ways. Firstly fuzzy mathematics induced by Lofti A Zadeh [31] with the commencement of fuzzy sets in 1965. This foundation represents a vagueness in everyday life. Subsequently many authors applied various form general topology of fuzzy sets and establish the approach of fuzzy space. Afterwards in 1975 two authors Kramosil and Michalek [12] introduced idea of fuzzy metric spaces. Author in 1988, has introduced extended fixed point theorem of banach and eldestien to fuzzy metric spaces in the perception of Kramosil and Michalek. In 1989 Grabiec [3] manifested an analog of the banach contraction theorem in fuzzy metric spaces using existing results. In his authentication, he employed a fuzzy version of Cauchy sequence. After few years Jachymski and Jozwik [11] in 2007 has given Non linear contractive conditions in fixed point theory. The subsistence of fixed point theorem for different sort of mappings in fuzzy metric spaces was examined by number of authors; see in some research papers like: Gregore and Sapena [4], Mihe [14]. Fixed point theory for contractive mappings in fuzzy metric spaces is meticulously associated with the fixed point theory for the similar kind of mappings in probabilistic metric spaces of menger type; see Hadzic [8], Sehgal and Bharucha-Reid [23], Schweizer *et al.* [21], Tardiff [25], Schweizer and Sklar [22], Qiu and Hong [17], Hong and Peng [9], Mohiuddine and Alotaibi [13], Wang *et al.* [28], Hong [10], Saadati *et al.* [18], and many others not specified in this paper.

In this paper, we establish a mapping and verified the fixed point theorem in complete fuzzy metric space. Following results we have studied for this conclusion. We inspired by the ideas of Wardowski [29], Wardowski and Dung [30], Dung and Hang [1], Piri and Kumam in 2014, Piri and Kumam [15]. They give the fixed point theorem by introducing F-contraction mapping, F-weak contraction mapping and F-suzuki contraction mapping in metric space. Next we inspired by the concept of Robert Plebaniak [16]. Then we generalized the above results of metric space in complete fuzzy metric space.

II. PRELIMINARIES

As Hussein Piri and Poom Kumam: They use some following result and proof theorem.

Definition 2.1: Let F be the family of all functions $F : R_+ \to R$ such that:

(2.1.1) F is strictly increasing, i.e. for all $x, y \in R_+$ such that $x < y, F(x) < F(y)$

(2.1.2) for each sequence $\{\alpha_n\}_{n=1}^{\infty}$ of positive numbers, $\lim_{n\to\infty} \alpha_n = 0$ if and only if $\lim_{n\to\infty} F(\alpha_n) = -\infty$

(2.1.3) there exists $k \in (0,1)$ such that $\lim_{\alpha\to 0} \alpha^k F(\alpha) = 0$.

Definition 2.2: [29] Let (X,d) be a metric space. A mapping $T: X \to X$ is stated to be an F-contraction on (X,d) if there exist $F \in \mathbf{F}$ and $\tau > 0$ such that, for all $x, y \in X$, $d(Tx,Ty) > 0 \Rightarrow \tau + F(d(Tx,Ty)) \le F(d(x,y))$. A new generalization of Banach contraction principle has been given by Wardowski as follows.

Theorem 2.3: [30] Let (X,d) be a complete metric space and let $T: X \to X$ be an F-contraction. Then T has a unique fixed point $x^* \in X$ and for every $x \in X$ the sequence $\{T^n x\}_{n=1}^{\infty}$ converges to x^*. In 2014, Wardowski and Dung [30] presented the idea of an F-weak contraction and proved a related fixed point theorem as follows.

Definition 2.4: [30] Let (X,d) be a metric space. A mapping $T: X \to X$ is said to be an F-weak contraction on (X,d) if there exist $F \in \mathbf{F}$ and $\tau > 0$ such that, for all $x, y \in X$, $d(Tx,Ty) > 0 \Rightarrow \tau + F(d(Tx,Ty)) \le F(M(x,y))$ in which

$$M(x,y) = max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty) + d(y,Tx)}{2}\right\}.$$

Theorem 2.5: [30] Let (X,d) be a complete metric space and let $T: X \to X$ be an F-weak contraction. If T or F is continuous, then T has a unique fixed point $x^* \in X$ and for every $x \in X$ the sequence $\{T^n x\}_{n=1}^{\infty}$ converges to x^*.

Definition 2.6: [3] Let (X,d) be a metric space. A mapping $T: X \to X$ is said to be a generalized F-contraction on (X,d) if there exist $F \in \mathbf{F}$ and $\tau > 0$ such that, for all $x, y \in X$, $d(Tx,Ty) > 0 \Rightarrow \tau + F(d(Tx,Ty)) \le F(N(x,y))$ in which

$$N(x,y) = max\left\{d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty) + d(y,Tx)}{2}, \frac{d(T^2x,x) + d(T^2x,Ty)}{2}, d(T^2x,Tx), d(T^2x,y), d(T^2x,Ty)\right\}$$

Theorem 2.7: [15] Let (X,d) be a complete metric space and let $T: X \to X$ be a generalized F-contraction mapping. If T or F is continuous, then T has a unique fixed point $x^* \in X$ and for every $x \in X$ the sequence $\{T^n x\}_{n=1}^{\infty}$ converges to x^*.

Theorem 2.8: [15] Let T be a self-mapping of a complete metric space X into itself. Suppose that there exists $F \in \mathbf{F}$ and $\tau > 0$ such that, for all $x, y \in X$ $d(Tx,Ty) > 0 \Rightarrow \tau + F(d(Tx,Ty)) \le F(d(x,y))$.

Then T has a unique fixed point $x^* \in X$ and for every $x \in X$ the sequence $\{T^n x\}_{n=1}^{\infty}$ converges to x^*.

Theorem 2.9: [15] Let T be a self-mapping of a complete metric space X into itself. Suppose that there exist $F \in \mathbf{F}$ and $\tau > 0$ such that, for all $x, y \in X$

$\frac{1}{2}d(x,Tx) < d(x,y) \Rightarrow \tau + F(d(Tx,Ty)) \le F(d(x,y))$. Then T has a unique fixed point $x^* \in X$ and for every $x \in X$ the sequence $\{T^n x\}_{n=1}^{\infty}$ converges to x^*.

Definition 2.10: [15] Let X be a nonempty set and $s \geq 1$ be a accorded actual number. A mapping $d : X \times X \to R_+$ is stated to be a b-metric if for all $x, y, z \in X$ the following conditions are satisfied:

(2.10.1) $d(x, y) = 0$ if and only if $x = y$;

(2.10.2) $d(x, y) = d(y, x)$;

(2.10.3) $d(x, z) \leq s[d(x, y) + d(y, z)]$.

In this case, the pair (X, d) is called a b-metric space (with constant s).

Theorem 2.11: Let (X, d) be a complete b-metric space and $T : X \to X$ be a generalized F-Suzuki-contraction. Then T has a unique fixed point $x^* \in X$ and for every $x \in X$ the sequence $\{T^n x\}_{n=1}^{\infty}$ converges to x^*.

As Robert Plebaniak : He give the theorem

Theorem 2.12: Let $(X, M, *)$ be a fuzzy metric space, and let N be a G-generalized fuzzy metric on X such that

$$\forall x, y \in X \left\{ \lim_{n\to\infty} N(x, y, t) = 1 \right\}$$

Let $T : X \to X$ be an N-G-contraction of Banach type, i.e., T is a mapping sufficing

(B1) $\exists k \in (0,1) \forall x, y \in X \forall t > 0 \{ N(T(x), T(y), kt) \geq N(x, y, t) \}$.

It is assume that a fuzzy metric space $(X, M, *)$ is N-G-complete. Then T has a unique fixed point $w \in X$, and for each $x \in X$, the sequence $(x_m = T^m(x_0) : x_0 = x, m \in N)$ is convergent to w. Moreover $, N(w, w, t) = 1, \forall\ t > 0.$

In this we generalized the above result.

A fuzzy metric space in the sense of Kramosil and Michalek (1975) is described as follows

Definition 2.13: [12] The 3-tuple $(X, M, *)$ is a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, and M is a fuzzy set in $X^2 \times [0, \infty)$ satisfying the following conditions:

(2.13.1) $\forall x, y \in X \{ M(x, y, 0) = 0 \}$;

(2.13.2) $\forall x, y \in X \{ \forall t > 0 (M(x, y, t) = 1) \Leftrightarrow x = y \}$;

(2.13.3) $\forall x, y \in X \forall t > 0 \{ M(x, y, t) = M(y, x, t) \}$;

(2.13.4) $\forall x, y, z \in X \forall t, s > 0 \{ M(x, z, t+s) \geq M(x, y, t) * M(y, z, s) \}$;

(2.13.5) $M(x, y, \cdot) : [0, \infty) \to [0, 1]$ is left-continuous, for all $x, y \in X$.

Then M is called a fuzzy metric on *X*.

Definition 2.14: (I) [12] A sequence $(x_m : m \in N)$ in X is Cauchy in Grabiec's sense (we say G-Cauchy) if $\forall t > 0 \forall p \in N \left\{ \lim_{m\to\infty} M(x_m, x_{m+p}, t) = 1 \right\}$.

(II) [3] A sequence $(x_m : m \in N)$ in X is convergent to $x \in X$ if $\forall t > 0 \left\{ \lim_{m\to\infty} M(x_m, x, t) = 1 \right\}$, *i.e.*, $\forall t > 0 \forall \varepsilon > 0 \exists m_0 \in N \forall m \geq m_0 \{ M(x_m, x, t) > 1 - \varepsilon \}$ Of course, since $* : [0,1] \times [0,1] \to [0,1]$ is continuous, by (2.13.4) it follows that the limit is uniquely determined.

(III) [3] A fuzzymetric space in which every G -Cauchy sequence is convergent is called complete in Grabiec's sense (G -complete for short).

Theorem 2.15: (Fuzzy Banach contraction theorem, Grabiec [3]). Let $(X, M, *)$ be a G complete fuzzy metric space such that

$$\forall x, y \in X \left\{ \lim_{t\to\infty} M(x, \mathrm{y}, t) = 1 \right\}$$

Let $T : X \to X$ be a mapping satisfying

(2.15.1) $\exists k \in (0,1) \forall x, y \in X \forall t > 0 \{ M(T(x), T(y), kt) \geq M(x, y, t) \}$.

Then T has a unique fixed point. Next, the idea of a fuzzy metric space is considered, which was brought to a knowledge by Schweizer in 1960.

Definition 2.16: A binary operation $*: [0,\ 1] \times [0,\ 1] \to [0,\ 1]$ is a continuous triangular norm (t-norm) if for all $a,\ b,\ c,\ e \in [0,\ 1]$ the succeeding conditions are fulfilled:

(i) $*$ is commutative and associative,
(ii) $a * 1 = a,$
(iii) $*$ is continuous, and
(iv) $a * b \leq c * e$ whenever $a \leq c$ *and* $b \leq e$.

The triplet $M(x,\ y, t)$ can be considered as the degree of proximity between x and y with respect to $t \geq 0$.

Lemma 2.17: For every $x,\ y \in X$, the mapping $M(x,\ y,\ .)$ is non-decreasing on $(0,\ \infty)$. Grabiec (1989) [3] expanded the fixed point theorem of Banach (1922) to fuzzy metric space in sense of Kramosil and Michalek (1975).

Theorem 2.18: (Grabiec, 1989) [3] Let $(X,\ M, *)$ be a complete fuzzy metric space gratifying

(i) $\lim_{t\to\infty} M(x,\ y, t) = 1$, and

(ii) $M(Fx,\ Fy,\ kt) \geq M(x,\ y, t),\ \forall x,\ y, \in X$, where $0 < k < 1$. Then F has a unique fixed point.

Then Vasuki (1998) [26] generalize Grabiecs result for common fixed point theorem for a sequence of mapping in a fuzzy metric space. Gregore and Sapena (2002) [4] proffered fixed point theorems for complete fuzzy metric space in the discernment of George and Veeramani (1994) [2] and also for Kramosil and Michaleks (1975) [12] fuzzy metric space that are complete in Grabeics sense. George and Veeramani (1994) [2] refined the abstraction of fuzzy metric space iniated by Kramosil and Michalek (1975)[12] with the assistance of t-norm and imparted the following definition.

Next, we recall the concept of a fuzzy metric space, which was started by George and Veeramani [2] in 1994

Definition 2.19: (George & Veeramani, 1994) [2] The triplet $(X,\ M, *)$ is stated to be fuzzy metric space if X is an arbitrary set, $*$ is continuous t-norm, and M is fuzzy set on $X^2 \times [0,\ \infty)$ satisfying the following conditions:

(2.13.1) $\forall x, y \in X \{ M(x, y, t) > 0 \} \forall t > 0$;

(2.13.2) $\forall x, y \in X \{ \forall t > 0 (M(x, y, t) = 1) \Leftrightarrow x = y \}$;

(2.13.3) $\forall x, y \in X \forall t > 0 \{ M(x, y, t) = M(y, x, t) \}$;

(2.13.4) $\forall x, y, z \in X \forall t, s > 0 \{ M(x, z, t+s) \geq M(x, y, t) * M(y, z, s) \}$;

(2.13.5) $M(x, y, \cdot) : [0, \infty) \to [0, 1]$ is left-continuous, for all $x, y \in X$.

Then M is considered to be a fuzzy metric on X.

By introducing this definition, they also achieved in introducing a Hausdorff topology on such fuzzy metric spaces which is widely employed these days by researchers in their particular area of research.

George and Veeramani (1994) have indicated that the definition of Cauchy sequence presented by Grabeic is weaker and hence it is indispensable to change that definition to get better outcome in fuzzy metric space. Consequently, some more metric fixed point results were generalized to fuzzy metric spaces by different authors like Subrahmanyam (1995)[24], Vasuki (1998)[26], Saini, Gupta, and Singh (2007)[19], Saini, Kumar, Gupta, and Singh (2008)[20], Vijayaraju (2009)[27], and Gupta and Mani (2014a, 2014b)[6-7]. Now some given important definitions and lemmas that are utilized in sequel.

Definition 2.20: [3] A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is said to be convergent to $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) = 1 \ \forall t > 0.$

Definition 2.21: [3] A sequence $\{x_n\}$ } in a fuzzy metric space $(X, M, *)$ is called Cauchy sequence if $\lim_{n\to\infty} M(xn+p, xn, t) = 1 \ \forall t > 0$ and each $p > 0.$

Definition 2.22: [3] A fuzzy metric space $(X, M, *)$ is considered to be complete if every Cauchy sequence in X converges in X.

Example : [5] Let (X, d) be a bounded metric space with $d(x, y) < k$ for all $x, y \in X$.

Let $g : R_+ \to (k, \infty)$ be an increasing continuous function. Define a function M as then $(X, M, *)$ is a fuzzy metric space on X where $*$ is a Lukasievicz t-norm, *i.e.* $*(a, b) = max\{a + b - 1, 0\}$.

Lemma 2.23: If there exists $k \in (0, 1)$ such that $M(x, y, kt) \geq M(x, y, t)$ for all $x, y \in X$ and $t \in (0, \infty)$, then $x = y$.

III. MAIN RESULT

Theorem 3.1: Let $(X, M, *)$ be a complete fuzzy metric space and $T : X \to X$ be a mapping defined as

$$M(Tx, Ty, kt) \geq M(x, y, t), \qquad \ldots(1)$$

where $k \in (0,1)$ and

$$M(x, y, t) = \max\left\{M(x, y, t), M(T^2x, y, t), \left(\frac{M(T^2x, Ty, t) + M(T^2x, Tx, t)}{2}\right), \left(\frac{M(T^2x, Ty, t) + M(Tx, x, t)}{2}\right), \left(\frac{M(Tx, y, t) + M(y, Ty, t)}{2}\right)\right\} \qquad \ldots(2)$$

Then T has a unique fixed point $x^* \in X$ and for every $x \in X$, the sequence $\{T^n x\}_{n=1}^{\infty}$ converges to x^*.

Proof: Take $x_0 = x \in X$. Let $x_n = Tx_{n-1} \ \forall n \in N.$

If there exists such that $M(x_n, Tx_n, t) = 1$ then $x = x_n$ becomes a fixed point of *T*, which completes the proof.

So by assumption of the theorem we have

$$M(Tx_n, Tx_{n+1}, kt) \geq M(x_n, x_{n+1}, t) \qquad \ldots(3)$$

By the above definition of mapping (2)

$$M(x_n, x_{n+1}, t) = \max\left\{M(x_n, x_{n+1}, t), M(T^2x_n, x_{n+1}, t), \left(\frac{M(T^2x_n, Tx_{n+1}, t) + M(T^2x_n, Tx_n, t)}{2}\right), \left(\frac{M(T^2x_n, Tx_{n+1}, t) + M(Tx_n, x_n, t)}{2}\right), \left(\frac{M(Tx_n, x_{n+1}, t) + M(x_{n+1}, Tx_{n+1}, t)}{2}\right)\right\}$$

$$= \max\left\{M(x_n, x_{n+1}, t), M(x_{n+2}, x_{n+1}, t), \left(\frac{M(x_{n+2}, x_{n+2}, t) + M(x_{n+2}, x_{n+1}, t)}{2}\right), \left(\frac{M(x_{n+2}, x_{n+2}, t) + M(x_{n+1}, x_n, t)}{2}\right), \left(\frac{M(x_{n+1}, x_{n+1}, t) + M(x_{n+1}, x_{n+2}, t)}{2}\right)\right\}$$

$$\geq \max\left\{M\left(x_n, x_{n+1}, t\right), M\left(x_{n+2}, x_{n+1}, t\right), M\left(x_{n+2}, x_{n+1}, t\right), M\left(x_{n+1}, x_n, t\right), M\left(x_{n+1}, x_{n+2}, t\right)\right\}$$

$$\geq \max\left\{M\left(x_n, x_{n+1}, t\right), M\left(x_{n+2}, x_{n+1}, t\right)\right\}$$

If $M\left(x_{n+2}, x_{n+1}, t\right) > M\left(x_n, x_{n+1}, t\right)$

Then from (3)

$$M\left(Tx_n, Tx_{n+1}, kt\right) = M\left(x_{n+1}, x_{n+2}, kt\right) \geq M\left(x_{n+1}, x_{n+2}, t\right)$$

Which is a contradiction, so we take

$$M\left(Tx_n, Tx_{n+1}, kt\right) \geq M\left(x_n, x_{n+1}, t\right) \qquad \ldots(4)$$

i.e. $M\left(x_{n+1}, x_{n+2}, kt\right) \geq M\left(x_n, x_{n+1}, t\right)$ Therefore $\left\{M\left(x_n, x_{n+1}, t\right)\right\}_{n\in N}$ is a non-negative decreasing sequence. Consequently we have

$$\lim_{n\to\infty} M\left(x_n, x_{n+1}, t\right) = 1 \qquad \ldots(5)$$

Now we affirm that $\left\{x_n\right\}_{n=1}^{\infty}$ is a Cauchy sequence.

We analyse that for each $x \in X$ we build a sequence $\left\{x_n\right\} \in X$ such that $Tx_n = Tx_{n+1}\ \forall n \in N$.

Let us take $x = x_{n-1}$ and $y = x_n$ in (1), then from (1) and (2), solve completely as in (3) we get

$$M\left(Tx_{n-1}, Tx_n, kt\right) \geq M\left(x_{n-1}, x_n, t\right)$$

Now by simple induction, for all n and $t > 0$

$$M\left(Tx_{n-1}, Tx_n, kt\right) \geq M\left(x_{n-1}, x_n, t\right)$$

$$= M\left(Tx_{n-2}, Tx_{n-1}, k\frac{t}{k}\right)$$

$$\geq M\left(x_{n-2}, x_{n-1}, \frac{t}{k}\right)$$

$$= M\left(Tx_{n-3}, Tx_{n-2}, k\frac{t}{k^2}\right)$$

$$\geq M\left(x_{n-3}, x_{n-2}, \frac{t}{k^2}\right)$$

……………………………proceeding this way, we get

$$M\left(x_n, x_{n+1}, kt\right) \geq M\left(x_0, x_1, \frac{t}{k^{n-1}}\right) \qquad ..(6)$$

Now for any positive numbers s, we have

$$M\left(x_n, x_{n+s}, t\right) \geq M\left(x_n, x_{n+1}, \frac{t}{s}\right) * \ldots\ldots\ldots\ldots * M\left(x_{n+p-1}, x_{n+p}, \frac{t}{s}\right)$$

Using (6), we get

$$M\left(x_n, x_{n+s}, t\right) \geq M\left(x_0, x_1, \frac{t}{sk^n}\right) * \ldots\ldots\ldots\ldots * M\left(x_0, x_1, \frac{t}{sk^n}\right)$$

Taking $\lim n \to \infty$

$$\lim_{n\to\infty} M\left(x_n, x_{n+s}, t\right) = 1 \qquad \ldots(7)$$

This implies that $\left\{x_n\right\}$ is a Cauchy sequence there exists a point $x^* \in X$ such that $\lim_{n\to\infty} x_n = x^*$

Next we see that for each $x \in X$ the sequence $\{x_n\}_{n=1}^{\infty}$ is convergent to a fixed point of T.

In above steps we have to proved that sequence $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence in X. By the completeness of X (using theorem 2.11) there exists $x^* \in X$ such that x_n is convergent to x^*

i.e. $\lim_{n\to\infty}\left(x_n, x^*, t\right) = 1$

Moreover $\forall t > 0\left[\lim_{n\to\infty}\left(x_n, x^*, t\right) = \lim_{n\to\infty}\left(x^*, x_n, t\right) = 1\right]$ …(8)

Consider using triangular inequality

$$\forall t > 0, n \in N\left\{M\left(x^*, Tx^*, t\right) \geq M\left(x^*, Tx_n, \frac{t}{2}\right) * M\left(Tx_n, Tx^*, k\frac{t}{2k}\right)\right\}$$

$$= M\left(x^*, x_{n+1}, \frac{t}{2}\right) * M\left(Tx_n, Tx^*, k\frac{t}{2k}\right)$$

Using (4)

$$\geq M\left(x^*, x_{n+1}, \frac{t}{2}\right) * M\left(x_n, x^*, \frac{t}{2k}\right) \text{ …(9)}$$

Again from (2)

$$M\left(x_n, x^*, \frac{t}{2k}\right) = \max\left\{M\left(x_n, x^*, \frac{t}{2k}\right), M\left(T^2x_n, x^*, \frac{t}{2k}\right), \frac{M\left(T^2x_n, Tx^*, \frac{t}{2k}\right) + M\left(T^2x_n, Tx_n, \frac{t}{2k}\right)}{2} + \frac{M\left(T^2x_n, Tx^*, \frac{t}{2k}\right) + M\left(Tx_n, x_n, \frac{t}{2k}\right)}{2} + \frac{M\left(Tx_n, x^*, \frac{t}{2k}\right) + M\left(x^*, Tx^*, \frac{t}{2k}\right)}{2}\right\}$$

$$= \max\left\{M\left(x_n, x^*, \frac{t}{2k}\right), M\left(x_{n+2}, x^*, \frac{t}{2k}\right), \frac{M\left(x_{n+2}, Tx^*, \frac{t}{2k}\right) + M\left(x_{n+2}, x_{n+1}, \frac{t}{2k}\right)}{2} + \frac{M\left(x_{n+2}, Tx^*, \frac{t}{2k}\right) + M\left(x_{n+1}, x_n, \frac{t}{2k}\right)}{2} + \frac{M\left(x_{n+1}, x^*, \frac{t}{2k}\right) + M\left(x^*, Tx^*, \frac{t}{2k}\right)}{2}\right\}$$

Letting $n \to \infty$ and using (8) then

$$M\left(x_n, x^*, \frac{t}{2k}\right) \geq M\left(x^*, Tx^*, \frac{t}{2k}\right)$$

Hence from (9)

$$M\left(x^*, Tx^*, t\right) \geq M\left(x^*, \mathrm{T}\,x^*, \frac{t}{2k}\right) * M\left(x^*, x_{n+1}{}^*, \frac{t}{2k}\right) \text{…(10)}$$

On taking $\lim n \to \infty$ and lemma 2.23

We conclude that $x^* = T_x{}^*$.

Hence x^* is a fixed point of *T*. Now we show that T has at most one fixed point.

Indeed if $x^*, y^* \in X$ are two fixed point of *T*. Such that $x^* \neq y^*$.

By the above proof there exists $y^* = Ty^*$ also we have $x^* = Tx^*$

Now we consider

$$1 \geq M\left(x^*, y^*, t\right)$$

$$= M\left(Tx^*, Ty^*, t\right) \geq M\left(x^*, y^*, \frac{t}{k}\right)$$

Where

$$M\left(x^*,y^*,\frac{t}{k}\right)=\max\left\{M\left(x^*,y^*,\frac{t}{k}\right),M\left(T^2x^*,y^*,\frac{t}{k}\right),\frac{M\left(T^2x^*,Ty^*,\frac{t}{k}\right)+M\left(T^2x^*,Tx^*,\frac{t}{k}\right)}{2},\frac{M\left(T^2x^*,Ty^*,\frac{t}{k}\right)+M\left(Tx^*,x^*,\frac{t}{k}\right)}{2},\frac{M\left(Tx^*,y^*,\frac{t}{k}\right)+M\left(y^*,Ty^*,\frac{t}{k}\right)}{2}\right\}$$

$$=\max\left\{M\left(x^*,y^*,\frac{t}{k}\right),M\left(x^*,y^*,\frac{t}{k}\right),\frac{M\left(x^*,y^*,\frac{t}{k}\right)+M\left(x^*,x^*,\frac{t}{k}\right)}{2},\frac{M\left(x^*,y^*,\frac{t}{k}\right)+M\left(x^*,x^*,\frac{t}{k}\right)}{2},\frac{M\left(x^*,y^*,\frac{t}{k}\right)+M\left(y^*,y^*,\frac{t}{k}\right)}{2}\right\}$$

$$=\max\left\{M\left(x^*,y^*,\frac{t}{k}\right),M\left(x^*,y^*,\frac{t}{k}\right),\frac{M\left(x^*,y^*,\frac{t}{k}\right)+1}{2},\frac{M\left(x^*,y^*,\frac{t}{k}\right)+1}{2},\frac{M\left(x^*,y^*,\frac{t}{k}\right)+1}{2}\right\}$$

$$\geq M\left(x^*,y^*,\frac{t}{k}\right)$$

i.e. $M\left(Tx^*,\mathrm{T}\,y^*,t\right)\geq M\left(x^*,y^*,\frac{t}{k}\right)$

So we have $x^*=y^*$. Thus x^* is unique fixed point of *T*. This complete the proof of the theorem.

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